An Implementation of Sparse Recovery via l_1 Minimization

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Motivation

- The problem of reconstructing, detecting, classifying, or estimating a signal from incoherent measurements is encountered in a variety of areas, e.g. optics, medical imaging, and data acquisition.
- Specific motivations include: reducing radiation exposure in MRIs; reducing cost of photon-detecting equipment; reducing data bandwidth, etc.

NARRATIVE

- Let α be some N-dimensional data.
- Let Ψ map α to some superposition, denoted by P.
- The support of P in Ψ is $K \leqslant N$ dimensions.
- On Ψ, take M *exact* measurements, m, of P via, say, Φ.
- Solve a minimization problem and then reconstruct P as $\hat{P}$.
- Track $\|P - \hat{P}\|$ as M and K vary.

PRINCIPAL THEORY

Let $\alpha \in \mathbb{R}^{21}$, $\Psi : [-1,1] \subset \mathbb{R}^{21} \to [-1,1] \subset \mathbb{R}^{21}$ of Vandermonde-type, $\Psi\alpha$ S-sparse, $\Phi : \mathbb{R}^M \to \mathbb{R}^N$, and define

$$P := \Psi\alpha \tag{1}$$

$$m := \Phi P \tag{2}$$

$$\hat{\alpha} := \min \|\alpha_0\|_1 \in \mathbb{R}^N \text{ s.t. } \Phi\Psi\alpha_0 = m \tag{3}$$

$$\hat{P} := \Psi\hat{\alpha} \tag{4}$$

$$K \sim S \tag{5}$$

then

$$M \geqslant K \log(N) \implies \|P - \hat{P}\| = 0 \tag{6}$$

as long as (2) is "uniformly uncertain" or "incoherent", i.e.

$$\frac{4M}{5N}\|\Psi\alpha\|_2^2 \leqslant \|\Phi\Psi\alpha\|_2^2 \leqslant \frac{5M}{4N}\|\Psi\alpha\|_2^2 \tag{7}$$

APPROACH

- Let entries in α, Ψ, and Φ vary randomly (save for the K-sparse control of α and the Vandermonde structure of Ψ), all restricted to $[-1, 1]$ for computational preconditioning.
- Note: Results can be generalized to the desired interval.
- Next, compute $\|P - \hat{P}\|$ as $K \leqslant 21$ and $M \leqslant 3 \cdot 21$ vary using $l - 1$ magic and Matlab's fmincon for Eqn. (3)
- Repeat 25 times to assess variational tendencies

ISSUE

- Even the 'preconditioned' Vandermonde operator, Ψ, and the uniformly uncertain measurement operator, Φ, always enjoy a very large condition number and their composition even more so.
- Although control of the condition number of Φ preserves its uniform uncertainty, control of the sample space generating Ψ has no observable effect on the conditioning of Ψ. **Moreover, directly controlling the condition number of Ψ always results in a false sample space (dim. > 21).**
- Hint: cond(Ψ) is always near 10^{21}. Hmmm...

Observation

- **Condition number of Vandermonde matrices is BOUNDED BELOW by an exponential for real entries.** Only under very strict conditions (namely when sample space is equidistant points on unit circle in $\mathbb{C}$) can Vandermonde matrix be well-conditioned – hence not a random sample space.
- Proof: Google "Arxiv Victor Pan How Bad Are Vandermonde Matrices"

Resolution

- Use matrix-free conjugate gradients version of minimization algorithms to avoid condition number issue and proceed as planned.
- Consider the fixed, well-conditioned case separately, where we force cond(Φ)$\equiv$ 1 and cond(Ψ)$\equiv$ 1 by exploiting singular value decomposition for the former and unit circle sample space $x_k = \cos(k\frac{2\pi}{21}) + i\sin(k\frac{2\pi}{21}), k = 0, ..., 20$ for latter.

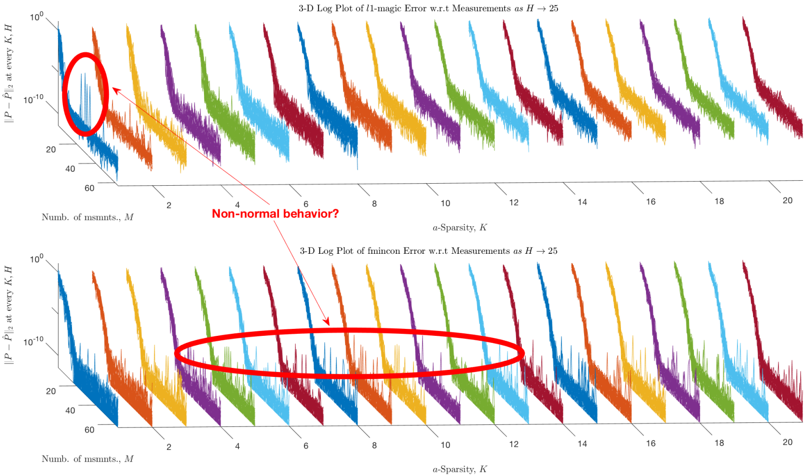
Initial Notes

Default Stopping Criteria

	Error tolerance	Max. iterations
Least squares guess	10^{-8}	500
$l-1$ magic	10^{-8}	500
fmincon	10^{-6}	3000

- Take error tolerance to be sufficient condition for algorithm's success
- Compute least squares/minimum energy guess for algorithm initialization
- Visual inspection of data suggests behavior may not be statistically normal. See *Figure 1*.

FIG. 1


3-D Log Plot of l1-magic Error w.r.t Measurements as $H \to 25$
10^0
10^{-10}
$\|P - \hat{P}\|_2$ at every K, H
20
40
60
2
4
6
8
10
12
14
16
18
20
Numb. of msmnts., M
Non-normal behavior?
a-Sparsity, K
3-D Log Plot of fmincon Error w.r.t Measurements as $H \to 25$
10^0
10^{-10}
$\|P - \hat{P}\|_2$ at every K, H
20
40
60
2
4
6
8
10
12
14
16
18
20
Numb. of msmnts., M
a-Sparsity, K

Resolution

Measure the standard deviation at every (M, K) with respect to H. See *Figure 2* and *Figure 3*.

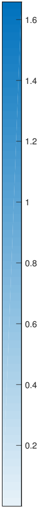
Fig. 2: Std. Dev. of $l-1$ magic at every (M, K)

	1	2	3	4	5	6	7	8	9	10	11
1	0.8476	0.6108	1.36	0.8635	1.112	1.272	1.061	1.251	2.102	1.336	1.362
2	0.4453	0.5707	0.8289	1.176	1.194	0.8764	1.172	0.7671	1.301	1.54	1.199
3	0.2452	0.3009	0.7579	0.457	0.3391	0.4874	0.5017	0.5641	1.076	1.093	0.7086
4	0.1325	0.388	0.3304	0.3298	0.341	0.3623	0.2937	0.3268	0.641	0.4844	0.6609
5	0.02647	0.1074	0.2185	0.241	0.4244	0.2277	0.3578	0.265	0.2849	0.1057	1.014
6	0.001678	0.06516	0.05048	0.09307	0.08274	0.1882	0.1054	0.07912	0.0892	0.1689	0.152
7	0.0001182	0.005629	0.01427	0.02423	0.06333	0.03901	0.05141	0.05273	0.05663	0.07425	0.1545
8	0.0001651	0.01165	0.03758	0.0111	0.006919	0.02218	0.02622	0.06621	0.03609	0.02299	0.03486
9	2.713e-06	0.002004	0.002209	0.005506	0.009437	0.01032	0.00961	0.0127	0.01124	0.03229	0.01922
10	2.48e-07	0.0005749	0.0002778	0.0004183	0.001108	0.003798	0.003177	0.003104	0.002536	0.01016	0.008154
11	9.376e-08	0.0001006	0.0001237	0.0004859	0.0005974	0.001562	0.001757	0.001665	0.003007	0.001392	0.001226
12	2.227e-07	4.078e-05	5.034e-05	0.0001087	0.001872	0.0002375	0.0004307	0.0007789	0.0004306	0.001122	0.002468
13	1.069e-08	9.703e-06	1.985e-05	3.447e-05	2.54e-05	9.97e-05	0.0003644	6.159e-05	0.0002344	0.0002616	8.82e-05
14	7.518e-09	1.981e-08	6.42e-06	1.042e-05	3.091e-05	1.462e-05	0.0001644	2.688e-05	5.743e-05	3.141e-05	7.738e-05
15	6.415e-09	1.346e-07	1.304e-06	3.907e-06	4.139e-06	2.416e-06	1.872e-05	2.528e-05	4.574e-05	1.253e-05	2.298e-05
16	4.658e-09	8.331e-07	4.384e-07	2.123e-07	7.788e-06	2.828e-06	1.3e-05	6.543e-06	7.154e-06	5.115e-06	1.117e-05
17	4.419e-09	6.04e-08	1.347e-07	3.672e-07	6.517e-07	2.743e-06	3.894e-06	6.587e-06	5.536e-06	1.106e-05	1.17e-05
18	3.106e-09	6.946e-09	1.273e-07	1.462e-06	8.559e-07	4.362e-07	8.427e-07	4.636e-06	7.962e-06	1.502e-05	9.295e-06
19	6.326e-09	8.625e-09	9.531e-07	6.509e-07	1.128e-06	3.518e-06	4.218e-06	4.098e-06	2.502e-06	2.892e-06	1.089e-05
20	5.277e-09	6.457e-09	1.559e-07	2.412e-07	1.434e-06	4.748e-06	1.307e-06	1.289e-06	2.848e-06	3.776e-06	4.76e-06
21	4.335e-09	1.183e-08	4.678e-08	6.014e-08	1.476e-06	7.285e-07	2.419e-06	2.324e-06	2.534e-05	6.251e-06	4.796e-06
22	6.716e-09	2.134e-07	1.884e-06	1.044e-06	1.053e-06	4.064e-06	1.297e-06	2.857e-06	2.15e-06	2.565e-06	8.276e-06
23	4.721e-09	1.371e-08	5.956e-07	4.668e-08	9.528e-07	7.24e-07	4.517e-06	2.246e-06	2.308e-05	5.201e-06	3.388e-06
24	1.132e-07	2.401e-08	6.123e-08	1.022e-07	3.102e-07	4.512e-06	2.097e-06	2.931e-06	6.071e-06	1.975e-06	3.091e-06
25	0.0007197	1.447e-08	6.144e-09	2.012e-06	8.204e-07	2.071e-07	2.964e-06	4.609e-06	1.318e-05	2.015e-06	7.68e-06
26	3.277e-09	3.334e-08	3.976e-07	2.175e-07	1.921e-05	9.524e-07	3.537e-06	1.453e-06	2.132e-06	3.419e-06	3.917e-06
27	3.994e-09	1.633e-08	2.238e-07	3.277e-07	5.054e-07	1.305e-06	1.324e-06	1.677e-06	4.39e-06	2.212e-06	2.883e-06
28	4.429e-09	5.438e-09	3.498e-08	4.116e-07	1.16e-06	1.184e-06	8.15e-07	9.04e-06	2.659e-06	1.888e-06	4.451e-06
29	0.002593	5.096e-08	4.097e-08	7.355e-07	6.255e-07	5.185e-07	8.495e-07	4.05e-06	1.882e-06	7.934e-06	2.055e-06
30	3.605e-09	1.858e-08	1.22e-07	6.923e-07	8.208e-07	2.525e-06	2.789e-06	2.375e-06	1.748e-05	3.521e-06	4.808e-06
31	0.0001983	5.019e-09	1.015e-08	3.982e-07	4.014e-07	3.093e-06	5.91e-06	3.264e-06	4.015e-06	6.459e-06	4.09e-06
32	3.696e-09	3.171e-08	2.708e-07	1.597e-07	1.935e-06	2.438e-06	1.341e-06	1.163e-06	1.918e-06	3.087e-06	1.322e-05
33	3.289e-09	5.379e-09	4.391e-07	1.69e-06	7.596e-07	4.01e-06	5.043e-06	3.944e-06	1.334e-06	1.067e-05	3.934e-06
34	0.0002568	1.398e-08	2.602e-07	4.634e-07	2.8e-06	2.305e-06	1.769e-06	5.567e-06	1.539e-06	3.53e-06	3.968e-06
35	3.141e-09	2.662e-07	3.778e-07	2.095e-07	1.933e-06	7.092e-07	6.467e-06	3.627e-06	1.821e-06	3.101e-06	3.294e-06
36	2.987e-09	5.408e-09	8.42e-09	3.811e-06	9.347e-07	1.067e-06	1.097e-06	1.201e-06	3.018e-06	3.207e-06	8.219e-06
37	2.357e-09	4.846e-09	8.707e-08	3.075e-07	5.12e-07	3.23e-06	1.371e-06	3.67e-06	1.04e-05	5.931e-06	6.883e-06
38	2.955e-09	6.273e-09	2.644e-08	7.779e-07	2.006e-07	2.439e-06	4.906e-07	4.512e-06	5.118e-06	5.889e-06	5.315e-06
39	3.984e-09	1.081e-08	2.348e-08	1.943e-07	5.461e-05	1.662e-06	5.07e-07	2.31e-06	2.718e-06	1.24e-05	3.113e-06
40	3.675e-09	1.055e-05	1.03e-07	9.6e-07	5.786e-07	4.887e-07	6.479e-06	2.737e-06	1.547e-06	9.579e-07	5.449e-06
41	6.643e-09	7.156e-09	2.085e-07	7.408e-07	1.012e-06	2.505e-06	3.467e-06	3.866e-06	4.214e-06	3.545e-06	3.01e-06
42	2.492e-09	3.628e-08	4.942e-07	3.358e-06	2.183e-06	7.63e-07	4.286e-06	1.571e-06	2.678e-06	3.689e-06	6.004e-06
43	2.688e-09	1.214e-08	6.778e-08	6.057e-07	1.674e-06	5.606e-07	3.617e-06	2.128e-06	9.763e-07	5.207e-06	4.694e-06
44	6.109e-09	5.097e-09	4.366e-08	2.062e-07	1.383e-06	7.437e-07	6.485e-07	3.521e-06	2.585e-06	2.839e-06	4.341e-06
45	2.66e-09	4.339e-09	7.262e-08	6.736e-08	3.883e-06	1.791e-06	2.369e-06	1.766e-06	5.56e-06	2.218e-06	3.959e-06
46	4.775e-09	1.352e-08	2.173e-08	5.805e-07	5.334e-06	2.746e-07	1.736e-06	6.851e-06	4.281e-06	3.998e-06	2.559e-06
47	1.99e-09	5.753e-07	4.89e-07	1.896e-06	1.093e-06	2.34e-06	5.023e-06	1.416e-06	3.996e-06	7.793e-06	8.046e-06
48	1.091e-06	1.736e-07	1.697e-06	2.149e-06	2.747e-07	1.74e-06	3.505e-06	2.223e-06	2.466e-06	5.437e-06	4.795e-06
49	3.344e-09	2.827e-08	6.076e-08	2.886e-07	8.757e-07	2.541e-06	4.242e-06	3.937e-06	3.106e-06	1.099e-05	1.451e-06
50	3.079e-09	1.041e-08	5.594e-07	2.224e-07	1.457e-06	6.551e-07	1.441e-06	3.19e-06	2.609e-06	2.728e-06	5.14e-06
51	5.592e-09	1.272e-08	8.198e-08	5.856e-06	3.195e-07	2.846e-06	4.577e-06	1.446e-06	4.859e-06	9.214e-06	2.782e-06
52	2.744e-09	1.12e-07	1.982e-07	1.916e-07	4.277e-06	2.915e-06	7.236e-06	5.095e-06	5.776e-06	1.087e-05	3.656e-06
53	4.006e-09	6.043e-08	3.67e-08	8.962e-07	1.6e-06	2.976e-06	1.988e-06	8.691e-06	1.01e-05	3.629e-06	6.907e-06
54	7.355e-09	1.19e-06	9.003e-07	2.24e-06	9.909e-07	1.716e-06	6.713e-06	5.145e-06	5.571e-06	1.213e-05	7.179e-06
55	3.66e-09	1.111e-07	2.683e-08	1.088e-06	3.551e-06	3.745e-06	3.686e-06	3.445e-06	7.661e-06	2.719e-06	2.801e-06
56	3.315e-09	4.146e-09	3.419e-07	4.394e-07	2.405e-06	1.137e-05	2.189e-06	1.769e-06	8.345e-06	6.463e-06	3.445e-06
57	2.966e-09	3.152e-09	7.121e-08	6.806e-07	2.74e-06	5.07e-07	1.705e-06	2.545e-06	2.572e-06	8.469e-06	4.829e-06
58	2.867e-09	5.266e-09	1.267e-06	4.126e-07	1.657e-06	9.23e-06	3.175e-06	6.589e-06	1.048e-05	6.406e-06	7.555e-06
59	1.457e-08	5.679e-08	2.118e-07	4.423e-06	9.007e-07	4.201e-06	3.601e-06	5.374e-06	3.629e-06	3.33e-06	5.379e-06
60	3.265e-09	5.551e-09	1.315e-07	5.506e-07	4.891e-07	3.945e-06	2.626e-06	2.134e-06	7.343e-06	4.239e-06	8.181e-06
61	2.849e-09	5.435e-07	5.719e-09	4.527e-07	2.107e-06	4.923e-06	2.007e-06	5.061e-06	1.003e-05	3.382e-06	7.989e-06
62	1.065e-08	6.463e-09	8.915e-08	4.091e-07	1.806e-06	1.427e-06	1.882e-06	5.476e-06	9.549e-06	2.888e-06	5.049e-06
63	2.611e-09	4.227e-09	1.96e-07	2.645e-07	1.863e-06	2.616e-06	1.939e-06	2.775e-06	2.254e-06	5.866e-06	8.404e-06

	12	13	14	15	16	17	18	19	20	21
1	2.115	1.251	1.445	1.635	1.22	1.67	1.306	1.631	1.474	2.47
2	1.195	2.044	1.602	2.279	1.988	1.102	0.9987	1.732	2.194	1.705
3	0.8835	0.8487	1.102	1.524	1.184	0.9908	1.18	1.392	1.393	1.304
4	0.8248	0.5747	0.5901	0.4053	0.5581	0.9901	0.6251	0.4314	0.7519	0.8243
5	0.2963	0.2627	0.2996	0.4504	0.4912	0.3004	1.14	0.3403	0.2391	0.4684
6	0.149	0.3208	0.1153	0.3407	0.09919	0.161	0.1238	0.3262	0.1833	0.5019
7	0.08898	0.1274	0.09927	0.06519	0.1382	0.1038	0.05926	0.1302	0.1518	0.1694
8	0.0603	0.03992	0.02507	0.05886	0.1299	0.09584	0.04699	0.2171	0.07212	0.05372
9	0.01032	0.01659	0.01528	0.01039	0.03758	0.02335	0.03718	0.02976	0.02684	0.01842
10	0.006032	0.00445	0.004795	0.0174	0.0195	0.01697	0.01174	0.01143	0.02882	0.005598
11	0.002527	0.003314	0.006645	0.002887	0.002269	0.00209	0.007984	0.002494	0.002617	0.004969
12	0.0006734	0.001005	0.0005527	0.001968	0.0007709	0.0008616	0.0007999	0.0009839	0.0009568	0.002565
13	0.0001916	0.0001979	0.000564	0.0003743	0.0006167	0.0008445	0.0003666	0.001054	0.00026	0.000413
14	8.325e-05	4.787e-05	7.932e-05	0.0001369	4.46e-05	0.00014	0.0002498	0.0001203	0.0001061	0.0002225
15	1.139e-05	5.25e-05	2.543e-05	2.798e-05	3.673e-05	3.018e-05	4.116e-05	8.508e-05	5.157e-05	2.853e-05
16	3.001e-05	1.14e-05	2.039e-05	4.265e-05	2.333e-05	2.307e-05	2.628e-05	2.009e-05	1.964e-05	7.104e-05
17	1.052e-05	4.356e-06	1.65e-05	6.348e-06	2.478e-05	1.876e-05	4.666e-05	8.457e-06	2.079e-05	1.145e-05
18	5.287e-06	5.555e-06	7.821e-06	9.558e-06	7.908e-06	1.786e-05	3.251e-05	8.81e-06	1.911e-05	2.145e-05
19	7.458e-06	1.879e-05	2.369e-05	1.287e-05	5.964e-06	9.321e-06	1.205e-05	2.763e-05	2.657e-05	9.878e-06
20	5.841e-06	3.019e-06	7.568e-06	3.084e-06	7.803e-06	4.816e-06	1.406e-05	7.817e-06	7.647e-06	1.34e-05
21	5.706e-06	1.201e-05	5.735e-06	9.595e-06	1.344e-05	2.923e-05	1.459e-05	9.263e-06	1.003e-05	1.002e-05
22	9.814e-06	3.148e-06	4.502e-06	8.297e-06	1.089e-05	1.35e-05	8.902e-06	4.33e-06	5.363e-06	1.457e-05
23	2.08e-06	4.837e-06	4.176e-06	9.413e-06	3.666e-06	5.463e-06	6.29e-06	9.725e-06	1.622e-05	7.82e-06
24	4.876e-06	6.001e-06	5.615e-06	9.465e-06	5.943e-06	5.394e-06	1.356e-05	7.892e-06	1.861e-05	2.366e-05
25	7.58e-06	4.291e-06	7.918e-06	2.587e-05	9.601e-06	1.01e-05	5.172e-06	1.792e-05	6.007e-06	5.565e-06
26	6.191e-06	1.234e-05	8e-06	7.204e-06	6.696e-06	8.253e-06	8.157e-06	7.988e-06	1.866e-05	6.059e-06
27	4.846e-06	7.485e-06	6.488e-06	1.093e-05	6.86e-06	1.046e-05	4.848e-06	9.613e-06	3.154e-06	9.03e-06
28	2.435e-06	7.733e-06	1.131e-05	1.171e-05	8.354e-06	6.092e-06	5.668e-06	1.821e-05	4.533e-06	1.677e-05
29	3.849e-06	7.825e-06	5.707e-06	6.397e-06	1.203e-05	3.833e-06	4.944e-06	9.092e-06	1.316e-05	5.849e-06
30	3.551e-06	9.081e-06	3.33e-06	4.933e-06	9.179e-06	2.664e-05	7.838e-06	4.923e-06	1.514e-05	1.574e-05
31	2.098e-05	7.515e-06	2.889e-06	7.899e-06	1.074e-05	9.528e-06	3.094e-05	7.209e-06	5.855e-06	1.201e-05
32	5.742e-06	1.489e-05	1.448e-05	1.277e-05	7.327e-06	6.445e-06	6.484e-06	5.107e-06	7.619e-06	3.368e-06
33	2.412e-05	2.671e-06	8.834e-06	3.504e-06	2.784e-06	8.795e-06	6.767e-06	5.42e-06	1.202e-05	9.423e-06
34	5.468e-06	2.339e-06	8.753e-06	4.49e-06	5.709e-06	5.311e-06	5.866e-06	7.528e-06	9.665e-06	1.261e-05
35	3.624e-06	4.046e-06	1.16e-05	2.579e-06	1.233e-05	1.167e-05	1.36e-05	2.692e-05	5.18e-06	1.129e-05
36	3.971e-06	7.299e-06	1.316e-05	1.36e-05	1.07e-05	5.859e-06	5.14e-06	1.559e-05	1.898e-05	1.327e-05
37	3.022e-05	7.871e-06	6.359e-06	2.934e-05	7.182e-06	1.595e-05	9.449e-06	8.08e-06	2.987e-06	3.929e-05
38	1.884e-06	5.961e-06	1.216e-05	1.298e-05	6.691e-06	1.577e-05	3.552e-06	3.828e-05	1.528e-05	1.126e-05
39	5.036e-06	7.772e-06	2.302e-06	1.151e-05	6.04e-06	7.218e-06	5.208e-06	5.192e-06	1.089e-05	5.173e-05
40	5.736e-06	6.503e-06	1.786e-05	1.562e-05	5.068e-06	3.27e-06	9.835e-06	1.784e-05	9.982e-05	8.428e-05
41	3.483e-06	4.302e-05	7.571e-06	1.375e-05	9.937e-06	3.022e-05	1.051e-05	1.277e-05	5.166e-06	3.128e-05
42	1.064e-05	3.882e-06	1.038e-05	6.521e-06	2.035e-05	1.523e-05	2.493e-05	1.933e-05	9.447e-06	1.77e-05
43	2.677e-06	4.943e-06	3.739e-05	3.195e-06	1.631e-05	9.058e-06	8.07e-06	7.696e-06	6.976e-06	1.47e-05
44	5.746e-06	1.061e-05	6.027e-06	1.1e-05	1.272e-05	1.318e-05	1.387e-05	9.803e-06	8.393e-06	1.112e-05
45	6.518e-06	9.312e-06	7.055e-06	3.905e-05	5.232e-06	7.349e-06	8.982e-06	1.754e-05	1.931e-05	1.206e-05
46	3.626e-06	1.017e-05	1.178e-05	1.686e-05	8.422e-06	4.664e-06	1.029e-05	1.886e-05	2.352e-05	9.166e-06
47	3.782e-06	3.208e-05	1.092e-05	8.719e-06	1.849e-05	9.978e-06	6.966e-06	1.606e-05	4.721e-06	7.059e-06
48	6.351e-06	1.349e-05	5.378e-06	1.567e-05	1.218e-05	1.195e-05	7.349e-06	1.481e-05	7.325e-05	1.211e-05
49	5.369e-06	1.311e-05	7.932e-06	5e-06	1.06e-05	9.212e-06	1.21e-05	1.084e-05	1.035e-05	3.163e-05
50	1.226e-05	5.28e-06	5.408e-06	2.406e-05	9.537e-06	4.695e-06	1.42e-05	1.121e-05	3.275e-05	6.282e-06
51	3.204e-06	1.565e-06	6.48e-06	8.304e-06	1.084e-05	1.685e-05	1.481e-05	3.398e-06	8.113e-06	3.4e-05
52	8.167e-06	6.722e-06	2.643e-05	1.284e-05	7.811e-06	8.415e-06	2.359e-05	7.987e-06	6.081e-06	1.788e-05
53	2.48e-06	8.386e-06	7.128e-06	1.008e-05	1.082e-05	6.702e-06	2.64e-05	1.048e-05	7.737e-06	9.907e-06
54	1.322e-05	8.445e-06	2e-05	1.254e-05	1.652e-05	3.101e-05	1.13e-05	9.066e-06	7.627e-06	3.851e-05
55	9.797e-06	2.445e-05	3.174e-06	6.31e-06	6.202e-06	3.63e-06	1.058e-05	9.861e-06	1.142e-05	1.217e-05
56	9.117e-06	1.232e-05	3.94e-06	1.646e-05	3.446e-06	1.794e-05	7.32e-06	1.606e-05	2.461e-05	2.142e-05
57	4.865e-06	2.093e-05	4.427e-06	9.984e-06	8.385e-06	1.974e-05	2.457e-05	1.684e-05	2.698e-05	7.847e-06
58	9.549e-06	3.5e-06	9.971e-06	1.348e-05	1.436e-05	4.762e-06	1.29e-05	1.756e-05	1.8e-05	8.801e-06
59	6.089e-06	1.666e-05	3.384e-06	1.691e-05	3.854e-06	2.512e-05	4.958e-06	1.369e-05	3.439e-05	7.114e-06
60	1.002e-05	9.411e-06	4.027e-05	1.597e-05	8.998e-06	4.466e-06	8.972e-06	1.316e-05	7.965e-06	1.154e-05
61	9.067e-06	1.774e-05	0.0001661	5.207e-06	1.118e-05	5.045e-06	3.207e-05	8.505e-06	4.417e-06	1.44e-05
62	1.871e-05	7.629e-06	5.081e-06	6.299e-06	1.203e-05	1.014e-05	2.698e-05	1.236e-05	9.185e-06	1.379e-05
63	1.399e-05	4.482e-06	1.84e-05	1.192e-05	1.702e-05	1.591e-05	2.183e-05	2.685e-05	2.047e-05	6.743e-06

FIG. 3: STD. DEV. OF FMINCON AT EVERY (M, K)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	0.62	0.4293	0.7455	0.6543	1.051	0.933	0.9736	0.807	1.279	0.8165	1.196	1.395	1.052	0.859	1.187	1.041	1.26	1.215	1.617	1.44	1.658
2	0.3596	0.3706	0.4208	0.9151	0.5908	0.8397	0.8635	0.8138	1.027	1.385	1.073	1.133	0.9492	0.7866	1.143	1.406	1.149	0.7423	1.394	1.037	1.216
3	0.2733	0.2644	0.6376	0.3926	0.2751	0.5442	0.4799	0.5558	0.8668	0.6284	0.6924	0.5749	0.6577	0.7859	1.296	0.8845	0.6115	0.8891	1.111	1.433	1.231
4	0.1062	0.1648	0.2203	0.2775	0.2399	0.2023	0.3457	0.2724	0.5067	0.4015	0.4852	0.4382	0.3955	0.4155	0.3582	0.6324	0.638	0.6574	0.4461	0.5137	0.6323
5	0.07978	0.06392	0.1783	0.1471	0.2502	0.1331	0.1985	0.3021	0.2406	0.1514	0.292	0.3673	0.2115	0.3618	0.2929	0.3697	0.3051	0.5403	0.3131	0.1879	0.5875
6	0.02241	0.03902	0.05784	0.1537	0.06805	0.115	0.1207	0.0858	0.1933	0.09875	0.1472	0.09393	0.2406	0.3099	0.1241	0.1026	0.07834	0.1166	0.1375	0.1803	0.2716
7	0.02656	0.02476	0.02261	0.02938	0.1041	0.03067	0.09923	0.04515	0.05715	0.05969	0.1408	0.03745	0.05247	0.07434	0.04028	0.07898	0.1438	0.07308	0.1063	0.1466	0.03305
8	0.008383	0.01761	0.04445	0.01474	0.009864	0.02206	0.0294	0.03918	0.02831	0.02277	0.0269	0.03109	0.04892	0.03287	0.03482	0.09494	0.06061	0.03239	0.1287	0.06367	0.05544
9	0.007863	0.01202	0.008396	0.007447	0.01299	0.01414	0.0104	0.01667	0.01044	0.06089	0.01878	0.009713	0.01524	0.01633	0.008669	0.02501	0.01766	0.02534	0.02935	0.0306	0.01186
10	0.005073	0.003138	0.002963	0.002846	0.002386	0.005483	0.006284	0.007334	0.004696	0.008521	0.005748	0.004138	0.003065	0.006419	0.01576	0.01631	0.008791	0.008245	0.006931	0.02625	0.006461
11	0.001085	0.0007698	0.001192	0.001358	0.00382	0.002649	0.002389	0.005881	0.002523	0.001019	0.001289	0.002482	0.002876	0.00453	0.001516	0.001702	0.002001	0.004286	0.001814	0.003348	0.003008
12	0.0004111	0.0003139	0.000821	0.0006424	0.001152	0.0003716	0.0006531	0.001825	0.0003498	0.0009363	0.001394	0.0005373	0.0007548	0.0008084	0.001256	0.0005733	0.0004749	0.001442	0.0008695	0.0004506	0.001678
13	0.0001325	8.348e-05	0.000173	0.0001168	0.0002366	0.0001822	0.0003546	0.000181	0.0003573	0.0002639	0.0001185	0.0002379	0.0001788	0.0006394	0.000356	0.0005652	0.0005063	0.000229	0.0005418	0.0003345	0.000269
14	3.104e-05	1.471e-05	2.543e-05	6.156e-05	5.416e-05	5.515e-05	0.0001917	4.893e-05	6.443e-05	9.328e-05	6.638e-05	5.199e-05	5.008e-05	0.0001063	9.546e-05	2.551e-05	8.096e-05	0.0001881	7.223e-05	8.183e-05	0.0002368
15	8.788e-06	8.805e-06	9.725e-06	3.023e-05	1.43e-05	4.221e-06	1.354e-05	1.036e-05	4.881e-05	6.343e-06	1.425e-05	9.161e-06	3.034e-05	1.493e-05	1.611e-05	1.3e-05	1.784e-05	2.147e-05	6.901e-05	3.166e-05	4.954e-05
16	4.084e-07	2.855e-06	3.297e-06	5.747e-06	7.113e-06	1.339e-05	3.855e-06	4.065e-06	6e-06	5.544e-06	4.764e-06	9.654e-06	5.82e-06	4.628e-06	1.129e-05	1.114e-05	6.285e-06	1.346e-05	8.668e-06	6.665e-06	9.337e-06
17	4.178e-07	2.824e-07	2.176e-07	3.224e-07	6.525e-07	5.975e-07	6.292e-07	8.337e-07	2.551e-07	1.24e-06	2.416e-06	3.194e-07	1.313e-06	1.163e-05	5.681e-07	7.742e-07	3.67e-07	3.221e-06	7.632e-07	1.308e-06	3.544e-06
18	1.318e-07	5.509e-08	1.51e-07	1.929e-07	6.161e-08	8.123e-08	1.982e-07	2.608e-07	2.381e-06	4.396e-07	5.598e-08	1.615e-07	3.716e-07	1.096e-07	2.161e-07	1.865e-07	3.029e-07	4.067e-07	2.055e-07	1.132e-06	1.511e-07
19	3.394e-08	2.431e-08	1.567e-08	1.485e-07	2.252e-08	3.235e-08	4.783e-08	3.024e-08	3.476e-08	1.049e-07	3.281e-08	8.235e-08	1.745e-07	1.331e-07	7.412e-08	2.371e-08	7.232e-08	8.176e-08	2.391e-07	9.111e-08	7.792e-08
20	1.041e-08	2.047e-08	3.61e-08	2.515e-08	3.351e-08	4.178e-08	2.157e-08	5.339e-08	1.86e-08	3.24e-08	5.923e-08	6.199e-08	2.572e-08	3.681e-08	7.299e-08	7.318e-08	6.012e-08	5.121e-08	1.035e-07	5.335e-08	4.694e-08
21	2.043e-09	1.696e-08	7.056e-10	1.555e-08	3.696e-09	3.595e-09	7.221e-10	1.492e-09	3.085e-09	6.219e-10	4.366e-09	1.739e-09	1.166e-09	1.028e-09	1.125e-08	2.722e-08	1.054e-09	5.285e-09	6.008e-09	4.292e-09	2.655e-09
22	3.13e-09	2.406e-09	8.812e-09	1.142e-09	1.23e-09	1.394e-09	7.611e-10	2.569e-09	5.712e-10	1.522e-09	1.161e-09	1.1e-09	1.074e-09	1.656e-09	1.627e-09	7.083e-10	7.178e-08	2.426e-09	1.433e-09	2.204e-09	5.208e-09
23	7.467e-09	6.933e-09	8.708e-10	5.452e-10	2.985e-09	9.881e-10	7.414e-10	3.26e-09	1.241e-09	1.371e-09	1.16e-09	1.197e-09	2.063e-09	1.011e-09	8.241e-10	5.751e-10	8.958e-10	7.341e-10	1.181e-09	9.073e-10	6.248e-10
24	2.791e-07	3.362e-09	5.716e-09	4.735e-10	4.687e-09	7.657e-10	7.061e-10	4.286e-10	1.485e-06	9.156e-10	1.09e-06	2.711e-09	7.731e-10	3.245e-09	3.415e-09	6.716e-10	5.197e-10	4.585e-10	5.327e-10	1.434e-09	1.03e-09
25	5.49e-09	9.499e-09	4.493e-10	4.884e-10	8.395e-09	5.14e-10	7.407e-09	6.657e-10	1.665e-09	4.53e-10	1.657e-10	3.493e-10	6.303e-10	6.08e-10	1.46e-09	1.157e-09	8.261e-10	4.709e-10	1.964e-08	2.139e-09	6.019e-10
26	6.662e-09	1.259e-08	1.97e-10	1.476e-08	5.3e-10	6.342e-10	2.665e-09	5.733e-10	1.59e-10	6.718e-10	6.438e-10	6.027e-10	4.639e-10	9.376e-10	1.08e-06	5.13e-10	6.525e-10	6.478e-10	3.261e-10	4.931e-10	4.653e-10
27	2.683e-09	1.344e-09	1.439e-09	8.997e-09	2.591e-10	4.433e-09	4.638e-10	3.653e-10	5.653e-10	1.291e-09	4.152e-10	5.05e-10	1.022e-09	6.111e-10	1.16e-08	4.251e-10	5.637e-10	2.581e-10	1.771e-08	3.091e-10	1.404e-09
28	2.164e-09	6.323e-09	4.154e-10	4.092e-10	5.516e-10	3.965e-10	3.897e-10	7.136e-09	5.899e-10	4.931e-10	5.822e-10	2.735e-10	2.657e-10	6.394e-10	7.556e-10	6.227e-10	8.626e-10	2.742e-10	8.257e-10	4.931e-09	5.033e-09
29	3.549e-09	9.65e-09	3.598e-10	6.713e-07	5.482e-10	2.999e-07	6.408e-10	7.526e-07	1.01e-08	3.805e-10	3.566e-10	9.37e-09	1.808e-09	8.055e-09	2.526e-10	7.19e-10	2.718e-06	3.682e-10	5.559e-10	2.184e-10	3.831e-10
30	4.134e-09	5.999e-10	7.267e-07	1.173e-06	3.156e-09	3.198e-10	2.054e-10	4.119e-10	3.501e-10	1.287e-09	6.642e-10	4.961e-10	2.176e-10	1.607e-08	4.087e-10	5.515e-10	6.873e-10	7.364e-09	6.372e-10	3.144e-10	3.808e-10
31	2.369e-09	3.094e-07	7.538e-09	7.935e-09	6.242e-09	4.71e-10	3.838e-10	2.52e-10	6.266e-07	5.031e-07	2.938e-10	2.697e-10	6.903e-10	3.67e-09	2.036e-10	2.806e-10	2.946e-10	6.243e-10	7.529e-08	1.266e-06	5.44e-10
32	4.636e-07	8.333e-09	5.508e-09	6.092e-07	6.174e-07	4.013e-10	5.957e-09	3.271e-10	1.906e-10	3.922e-10	4.008e-10	3.345e-09	7.496e-10	2.277e-06	5.287e-10	6.178e-10	3.285e-10	4.714e-10	1.399e-06	7.565e-10	1.807e-10
33	4.645e-09	3.597e-09	4.11e-09	4.09e-09	1.362e-09	3.086e-10	9.913e-10	4.301e-10	6.262e-07	1.179e-06	2.573e-10	7.808e-09	2.844e-06	2.829e-10	1.504e-08	5.144e-10	8.772e-09	6.988e-10	3.245e-10	4.548e-10	9.022e-10
34	3.582e-09	2.262e-09	3.494e-09	1.622e-10	1.159e-06	7.022e-10	2.147e-06	5.477e-10	2.62e-10	2.892e-10	2.617e-10	3.73e-09	2.987e-09	2.103e-10	2.602e-10	2.564e-10	7.245e-10	1.217e-08	5.677e-10	9.798e-07	2.37e-10
35	2.669e-09	7.449e-08	3.714e-09	6.438e-07	1.209e-10	2.042e-10	6.275e-07	9.1e-07	2.372e-10	1.376e-06	1.116e-08	5.557e-10	9.062e-10	2.36e-10	4.528e-10	4.811e-10	1.297e-09	7.414e-07	2.194e-08	3.604e-10	6.122e-10
36	6.985e-09	2.053e-09	3.225e-10	1.979e-10	1.8e-06	2.318e-10	2.903e-10	3.763e-10	4.573e-07	2.878e-10	2.077e-10	1.476e-10	2.605e-10	5.754e-09	1.32e-06	2.69e-10	7.322e-10	1.575e-07	3.828e-10	3.415e-09	3.018e-10
37	1.756e-09	5.146e-10	2.473e-10	5.073e-09	2.622e-10	6.577e-09	3.041e-10	1.013e-10	5.25e-10	4.028e-10	1.094e-09	1.56e-10	6.162e-10	4.48e-10	7.361e-10	6.613e-10	4.22e-07	1.062e-08	2.563e-10	2.351e-09	4.535e-10
38	4.243e-09	2.302e-09	4.546e-09	1.111e-08	8.92e-07	3.662e-10	2.419e-10	2.404e-10	9.753e-11	2.493e-10	3.384e-10	2.539e-10	3.654e-08	8.888e-09	2.955e-10	3.022e-07	4.585e-10	4.002e-10	3.238e-10	2.375e-10	1.466e-08
39	8.536e-07	7.969e-09	1.362e-10	4.108e-10	8.967e-09	1.58e-08	3.939e-10	1.623e-09	3.009e-10	2.44e-09	2.724e-10	8.425e-09	1.239e-10	8.412e-09	3.536e-10	3.15e-10	4.681e-10	4.386e-07	3.938e-10	1.987e-08	6.808e-10
40	3.913e-09	1.54e-09	1.519e-10	2.014e-09	2.94e-10	5.335e-10	1.525e-10	3.41e-10	1.663e-10	1.279e-08	1.889e-10	5.539e-10	3.321e-10	3.26e-10	1.355e-10	1.716e-10	2.042e-10	2.371e-10	4.083e-10	3.605e-10	1.635e-10
41	9.438e-09	2.478e-09	2.393e-09	1.01e-10	2.62e-10	7.786e-08	7.551e-07	3.126e-07	4.495e-09	2.729e-07	1.276e-10	1.19e-10	1.111e-06	3.065e-07	3.852e-10	1.191e-09	2.545e-10	2.063e-09	2.805e-10	9.788e-07	3.486e-10
42	2.786e-09	1.145e-09	9.72e-07	1.503e-07	2.214e-10	2.383e-10	3.211e-10	1.188e-09	1.009e-10	2.541e-10	4.288e-09	2.715e-09	2.032e-10	2.69e-10	1.765e-10	2.376e-10	3.123e-06	1.976e-10	4.392e-10	2.22e-10	3.981e-10
43	8.85e-09	1.225e-08	2.674e-09	5.547e-09	3.498e-09	1.05e-08	1.995e-10	2.222e-10	1.567e-10	2.688e-10	1.695e-06	1.362e-10	7.542e-09	3.351e-10	4.116e-10	1.738e-10	3.182e-10	2.328e-10	3.59e-10	2.334e-10	5.857e-10
44	1.489e-09	3.034e-09	1.314e-06	2.953e-10	2.668e-10	1.753e-10	2.413e-10	3.071e-09	3.411e-07	1.17e-08	1.345e-10	2.033e-10	2.269e-10	3.94e-10	1.565e-06	1.843e-10	7.63e-09	1.835e-10	2.582e-10	2.579e-10	8.735e-09
45	3.71e-09	1.784e-09	1.744e-10	1.133e-06	2.918e-07	3.981e-10	1.93e-06	2.995e-10	1.442e-10	9.331e-11	2.856e-10	1.036e-09	4.469e-07	1.283e-10	2.888e-10	1.469e-08	3.476e-10	1.386e-10	4.469e-07	2.755e-10	2.957e-10
46	1.335e-09	6.838e-09	1.353e-09	5.003e-09	1.949e-10	2.286e-10	2.477e-10	3.448e-10	3.121e-10	6.319e-10	3.846e-10	3.02e-10	2.898e-10	6.694e-09	3.534e-10	3.243e-09	1.133e-10	1e-08	1.301e-06	1.82e-09	4.207e-09
47	3.007e-09	1.853e-09	9.549e-11	6.562e-07	8.127e-09	8.551e-10	1.164e-08	2.957e-09	6.14e-09	2.535e-10	2.152e-10	1.804e-10	3.52e-10	2.487e-10	3.259e-07	2.021e-10	8.572e-09	1.488e-08	1.543e-06	2.644e-10	2.347e-10
48	6.764e-09	5.638e-09	7.872e-09	8.342e-09	1.677e-10	2.307e-10	2.848e-10	1.194e-06	1.829e-06	1.959e-10	2.015e-10	1.716e-10	2.955e-10	1.972e-10	1.179e-06	7.079e-07	7.534e-10	1.89e-10	2.091e-10	7.552e-07	4.04e-10
49	3.677e-09	9.69e-09	1.232e-10	1.034e-10	1.47e-09	6.979e-07	4.733e-09	3.089e-07	2.201e-09	2.832e-10	2.124e-10	2.747e-10	9.956e-09	1.87e-10	2.227e-10	2.602e-06	1.919e-10	2.078e-10	2.283e-10	8.255e-10	1.055e-07
50	2.227e-09	1.657e-09	1.832e-10	3.799e-09	3.2e-10	7.979e-11	1.232e-10	2.256e-10	1.329e-10	8.951e-07	1.458e-10	1.629e-06	1.422e-10	1.767e-10	5.355e-07	1.84e-08	4.878e-09	1.314e-10	3.42e-10	1.572e-10	4.058e-10
51	7.706e-09	4.584e-07	9.301e-08	3.444e-09	7.638e-07	2.317e-10	1.201e-10	1.24e-10	1.818e-10	2.219e-10	2.339e-10	3.293e-07	1.059e-06	2.697e-10	8.005e-07	1.183e-06	6.196e-09	2.621e-07	2.25e-09	2.807e-10	9.465e-11
52	8.037e-08	3.992e-09	3.803e-11	9.35e-09	3.739e-09	1.721e-10	8.834e-10	1.27e-10	3.754e-10	6.725e-09	3.103e-10	2.188e-09	1.682e-10	6.494e-09	2.492e-10	1.195e-08	1.304e-08	1.536e-06	1.396e-06	6.203e-10	1.007e-06
53	8.344e-09	1.008e-08	7.331e-10	1.634e-07	3.005e-09	5.138e-09	1.115e-10	2.899e-07	1.053e-08	1.897e-10	2.49e-10	2.013e-10	1.332e-10	4e-09	8.979e-09	1.591e-10	1.252e-10	2.335e-10	1.364e-10	3.524e-10	3.188e-10
54	2.806e-09	3.709e-09	5.936e-07	9.878e-11	1.084e-10	5.952e-11	1.701e-10	1.886e-10	2.67e-10	2.445e-10	1.203e-10	8.364e-09	1.078e-10	4.426e-10	4.604e-07	1.669e-10	1.2e-10	1.686e-10	2.435e-09	4.016e-10	1.2e-08
55	5.536e-09	3.038e-09	6.575e-07	1.654e-09	1.469e-10	1.615e-10	3.649e-10	1.105e-08	1.267e-10	4.426e-10	1.647e-10	4.293e-07	1.286e-10	1.553e-10	1.42e-10	1.579e-10	1.534e-10	1.331e-10	2.714e-10	1.118e-10	3.218e-10
56	4.726e-09	6.323e-07	2.095e-07	1.408e-10	8.471e-10	4.397e-07	2.634e-10	1.998e-10	1.265e-10	2.009e-10	1.668e-10	9.223e-09	1.894e-10	8.415e-07	2.575e-10	2.038e-10	1.204e-06	1.46e-10	4.982e-09	9.285e-11	1.216e-10
57	2.464e-09	3.067e-09	6.896e-07	3.53e-10	2.579e-09	4.078e-09	1.897e-10	7.175e-10	1.707e-10	1.25e-10	2.167e-10	7.189e-09	1.834e-10	2.852e-10	2.481e-10	1.307e-08	1.206e-08	2.849e-10	3.317e-09	4.197e-07	3.118e-07
58	2.54e-09	5.246e-09	2.687e-07	4.521e-09	2.151e-10	5.74e-09	5.667e-09	1.564e-10	5.248e-11	8.624e-09	2.504e-10	2.552e-10	1.64e-10	2.087e-10	4.452e-09	1.989e-10	1.42e-09	4.038e-07	9.801e-11	1.12e-10	2.584e-10
59	2.343e-09	1.751e-10	8.351e-09	5.284e-10	4.934e-07	2.762e-10	6.935e-09	6.94e-09	1.814e-10	1.131e-10	6.792e-10	1.758e-10	2.296e-06	2.51e-10	3.887e-09	2.627e-10	3.422e-10	9.542e-07	6.869e-09	2.2e-08	2.041e-10
60	4.671e-09	6.314e-11	5.532e-09	9.729e-11	3.358e-09	1.376e-10	4.201e-09	1.517e-10	1.15e-09	8.132e-11	1.753e-10	1.383e-10	9.104e-09	7.012e-09	1.956e-10	5.133e-07	5.473e-09	2.799e-09	1.626e-10	2.056e-10	1.506e-08
61	1.838e-09	4.856e-09	4.198e-09	1.564e-10	1.235e-10	9.675e-07	1.338e-10	1.175e-06	1.74e-06	8.217e-09	1.673e-10	1.04e-10	2.507e-08	4.988e-10	1.717e-06	1.349e-10	1.827e-08	1.922e-08	3.155e-10	2.188e-10	5.139e-07
62	2.569e-09	2.272e-09	1.735e-10	1.919e-09	4.229e-09	1.296e-10	1.249e-10	1.583e-10	9.193e-07	1.116e-10	3.329e-10	2.169e-07	1.15e-10	2.1e-10	2.348e-10	1.33e-06	1.798e-10	4.553e-09	2.238e-10	2.446e-10	1.872e-10
63	4.476e-09	1.414e-07	7.262e-09	8.753e-10	1.221e-10	3.861e-09	1.322e-10	8.262e-07	8.163e-11	1.764e-10	1.129e-08	1.753e-10	1.279e-10	2.01e-10	1.404e-10	2.232e-10	5.442e-09	3.279e-10	2.362e-10	1.857e-10	5.363e-09



REMARKS

- Error measures are reasonably normal over space of hyperiterations (i.e.$\leqslant 2$)
- fmincon enjoys a tighter error distribution, likely due to default iterative stopping criterion
- Therefore, we may proceed by considering the mean errors at every (M, K) as reasonably representative

RESULTS

For $l-1$ magic, successful (M, K) pairs occur about when

$$\frac{M}{K\log(21)} \geqslant 2.2 \tag{8}$$

excluding all $(M=1, K)$ **and all** $(M, K>5)$. For fmincon, successful (M, K) almost always occur when

$$M \geqslant 17 \tag{9}$$

and admits no reasonable inequality bound like $l-1$ magic, with similar $M=1$ behavior. See *Figure 4*, *Figure 5*, and *Figure 6*.

FIG. 4

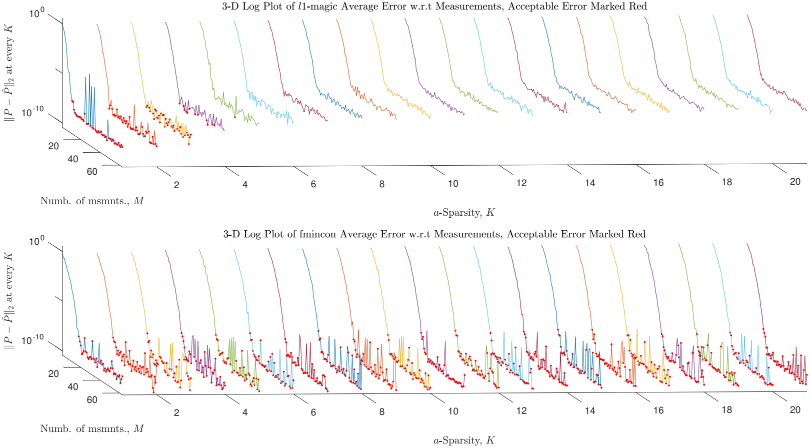

3-D Log Plot of $l1$-magic Average Error w.r.t Measurements, Acceptable Error Marked Red
10^0
10^{-10}
$\|P - \hat{P}\|_2$ at every K
20
40
60
Numb. of msmnts., M
2
4
6
8
10
12
14
16
18
20
a-Sparsity, K
3-D Log Plot of fmincon Average Error w.r.t Measurements, Acceptable Error Marked Red
10^0
10^{-10}
$\|P - \hat{P}\|_2$ at every K
20
40
60
Numb. of msmnts., M
2
4
6
8
10
12
14
16
18
20
a-Sparsity, K

FIG 5: SUCCESSFUL $\frac{M}{K\log(21)}$ FOR $l-1$ MAGIC

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	3.613	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	3.942	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	4.27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	4.598	2.299	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	4.927	2.463	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	5.255	0	0	1.314	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	5.584	2.792	1.861	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	5.912	2.956	1.971	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	6.241	3.12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	6.569	3.285	2.19	1.642	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	6.898	3.449	2.299	1.724	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	7.226	3.613	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	7.555	3.777	0	1.889	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	7.883	3.942	2.628	1.971	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	4.106	2.737	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	8.54	4.27	2.847	2.135	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	8.868	4.434	2.956	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	9.197	4.598	3.066	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	4.763	3.175	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	9.854	4.927	3.285	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	5.091	3.394	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	10.51	5.255	3.504	2.628	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	10.84	5.42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	5.584	3.723	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	11.5	5.748	3.832	2.874	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	11.82	5.912	3.942	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	12.15	6.076	4.051	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	12.48	6.241	4.16	0	2.496	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	12.81	6.405	4.27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	13.14	0	4.379	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	13.47	6.733	4.489	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	13.8	6.898	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	14.12	7.062	4.708	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	14.45	7.226	4.817	3.613	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	14.78	7.39	4.927	3.695	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	15.11	7.555	5.036	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	15.44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	7.883	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	16.09	8.047	5.365	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	16.42	8.211	0	4.106	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	16.75	8.376	5.584	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	17.08	8.54	5.693	4.27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	17.41	8.704	5.803	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	17.74	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	18.07	9.033	6.022	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	18.39	9.197	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	18.72	9.361	6.241	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	19.05	9.525	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	19.38	9.69	6.46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	19.71	9.854	6.569	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	20.04	0	6.679	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	20.36	10.18	6.788	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	20.69	10.35	6.898	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

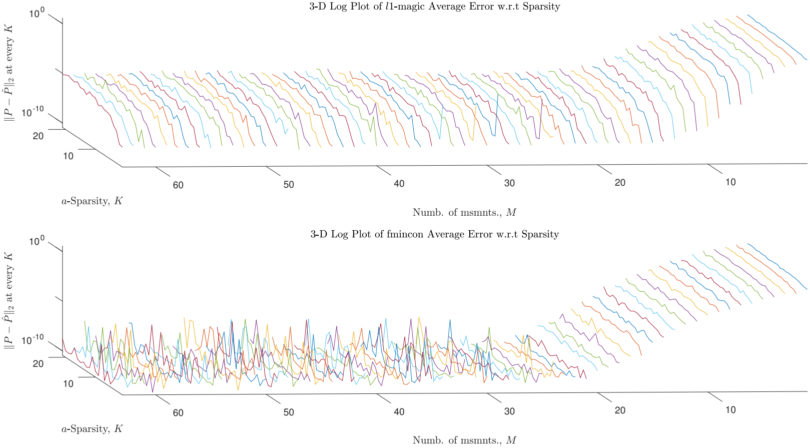
FIG. 6: SUCCESSFUL $\frac{M}{K\log(21)}$ FOR FMINCON

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	5.912	2.956	1.971	1.478	1.182	0.9854	0	0.739	0	0	0.5375	0.4927	0	0.4223	0.3942	0.3695	0	0	0	0	0.2815
19	6.241	3.12	2.08	1.56	1.248	1.04	0.8915	0.7801	0.6934	0.6241	0.5673	0.5201	0.4801	0.4458	0.416	0.39	0.3671	0.3467	0	0.312	0.2972
20	6.569	3.285	2.19	1.642	1.314	1.095	0.9385	0.8211	0.7299	0.6569	0.5972	0.5474	0.5053	0.4692	0.4379	0.4106	0.3864	0.365	0.3457	0.3285	0.3128
21	6.898	3.449	2.299	1.724	1.38	1.15	0.9854	0.8622	0.7664	0.6898	0.6271	0.5748	0.5306	0.4927	0.4598	0.4311	0.4057	0.3832	0.363	0.3449	0.3285
22	7.226	3.613	2.409	1.807	1.445	1.204	1.032	0.9033	0.8029	0.7226	0.6569	0.6022	0.5559	0.5161	0.4817	0.4516	0.4251	0.4014	0.3803	0.3613	0.3441
23	7.555	3.777	2.518	1.889	1.511	1.259	1.079	0.9443	0.8394	0.7555	0.6868	0.6295	0.5811	0.5396	0.5036	0.4722	0.4444	0.4197	0.3976	0.3777	0.3597
24	7.883	3.942	2.628	1.971	1.577	1.314	1.126	0.9854	0	0.7883	0	0.6569	0.6064	0.5631	0.5255	0.4927	0.4637	0.4379	0.4149	0.3942	0.3754
25	8.211	4.106	2.737	2.053	1.642	1.369	1.173	1.026	0.9124	0.8211	0.7465	0.6843	0.6317	0.5865	0.5474	0.5132	0.483	0.4562	0.4322	0.4106	0.391
26	8.54	4.27	2.847	2.135	1.708	1.423	1.22	1.067	0.9489	0.854	0.7764	0.7117	0.6569	0.61	0	0.5337	0.5023	0.4744	0.4495	0.427	0.4067
27	8.868	4.434	2.956	2.217	1.774	1.478	1.267	1.109	0.9854	0.8868	0.8062	0.739	0.6822	0.6335	0.5912	0.5543	0.5217	0.4927	0.4668	0.4434	0.4223
28	9.197	4.598	3.066	2.299	1.839	1.533	1.314	1.15	1.022	0.9197	0.8361	0.7664	0.7074	0.6569	0.6131	0.5748	0.541	0.5109	0.484	0.4598	0.4379
29	9.525	4.763	3.175	0	1.905	1.588	1.361	0	1.058	0.9525	0.8659	0.7938	0.7327	0.6804	0.635	0.5953	0	0.5292	0.5013	0.4763	0.4536
30	9.854	4.927	0	0	1.971	1.642	1.408	1.232	1.095	0.9854	0.8958	0.8211	0.758	0.7038	0.6569	0.6159	0.5796	0.5474	0.5186	0.4927	0.4692
31	10.18	5.091	3.394	2.546	2.036	1.697	1.455	1.273	0	0	0.9257	0.8485	0.7832	0.7273	0.6788	0.6364	0.599	0.5657	0.5359	0	0.4849
32	10.51	5.255	3.504	0	0	1.752	1.502	1.314	1.168	1.051	0.9555	0.8759	0.8085	0	0.7007	0.6569	0.6183	0.5839	0	0.5255	0.5005
33	10.84	5.42	3.613	2.71	2.168	1.807	1.548	1.355	0	0	0.9854	0.9033	0	0.7742	0.7226	0.6774	0.6376	0.6022	0.5705	0.542	0.5161
34	11.17	5.584	3.723	2.792	0	1.861	0	1.396	1.241	1.117	1.015	0.9306	0.859	0.7977	0.7445	0.698	0.6569	0.6204	0.5878	0	0.5318
35	11.5	5.748	3.832	0	2.299	1.916	0	0	1.277	0	1.045	0.958	0.8843	0.8211	0.7664	0.7185	0.6762	0	0.6051	0.5748	0.5474
36	11.82	5.912	3.942	2.956	0	1.971	1.689	1.478	1.314	1.182	1.075	0.9854	0.9096	0.8446	0	0.739	0.6956	0.6569	0.6223	0.5912	0.5631
37	12.15	6.076	4.051	3.038	2.431	2.025	1.736	1.519	1.35	1.215	1.105	1.013	0.9348	0.8681	0.8102	0.7596	0.7149	0.6752	0.6396	0.6076	0.5787
38	12.48	6.241	4.16	3.12	0	2.08	1.783	1.56	1.387	1.248	1.135	1.04	0.9601	0.8915	0.8321	0.7801	0.7342	0.6934	0.6569	0.6241	0.5944
39	0	6.405	4.27	3.202	2.562	2.135	1.83	1.601	1.423	1.281	1.165	1.067	0.9854	0.915	0.854	0.8006	0.7535	0.7117	0.6742	0.6405	0.61
40	13.14	6.569	4.379	3.285	2.628	2.19	1.877	1.642	1.46	1.314	1.194	1.095	1.011	0.9385	0.8759	0.8211	0.7728	0.7299	0.6915	0.6569	0.6256
41	13.47	6.733	4.489	3.367	2.693	2.244	0	1.683	1.496	1.347	1.224	1.122	0	0.9619	0.8978	0.8417	0.7922	0.7482	0.7088	0	0.6413
42	13.8	6.898	0	3.449	2.759	2.299	1.971	1.724	1.533	1.38	1.254	1.15	1.061	0.9854	0.9197	0.8622	0	0.7664	0.7261	0.6898	0.6569
43	14.12	7.062	4.708	3.531	2.825	2.354	2.018	1.765	1.569	1.412	0	1.177	1.086	1.009	0.9416	0.8827	0.8308	0.7847	0.7434	0.7062	0.6726
44	14.45	7.226	0	3.613	2.89	2.409	2.065	1.807	1.606	1.445	1.314	1.204	1.112	1.032	0	0.9033	0.8501	0.8029	0.7606	0.7226	0.6882
45	14.78	7.39	4.927	0	2.956	2.463	0	1.848	1.642	1.478	1.344	1.232	1.137	1.056	0.9854	0.9238	0.8694	0.8211	0.7779	0.739	0.7038
46	15.11	7.555	5.036	3.777	3.022	2.518	2.158	1.889	1.679	1.511	1.374	1.259	1.162	1.079	1.007	0.9443	0.8888	0.8394	0	0.7555	0.7195
47	15.44	7.719	5.146	0	3.088	2.573	2.205	1.93	1.715	1.544	1.403	1.286	1.188	1.103	1.029	0.9648	0.9081	0.8576	0	0.7719	0.7351
48	15.77	7.883	5.255	3.942	3.153	2.628	2.252	0	0	1.577	1.433	1.314	1.213	1.126	0	0	0.9274	0.8759	0.8298	0	0.7508
49	16.09	8.047	5.365	4.024	3.219	0	2.299	2.012	1.788	1.609	1.463	1.341	1.238	1.15	1.073	0	0.9467	0.8941	0.8471	0.8047	0.7664
50	16.42	8.211	5.474	4.106	3.285	2.737	2.346	2.053	1.825	0	1.493	0	1.263	1.173	0	1.026	0.9661	0.9124	0.8644	0.8211	0.782
51	16.75	8.376	5.584	4.188	0	2.792	2.393	2.094	1.861	1.675	1.523	1.396	0	1.197	0	0	0.9854	0.9306	0.8817	0.8376	0.7977
52	17.08	8.54	5.693	4.27	3.416	2.847	2.44	2.135	1.898	1.708	1.553	1.423	1.314	1.22	1.139	1.067	1.005	0	0	0.854	0
53	17.41	8.704	5.803	4.352	3.482	2.901	2.487	2.176	1.934	1.741	1.583	1.451	1.339	1.243	1.161	1.088	1.024	0.9671	0.9162	0.8704	0.829
54	17.74	8.868	0	4.434	3.547	2.956	2.534	2.217	1.971	1.774	1.612	1.478	1.364	1.267	1.182	1.109	1.043	0.9854	0.9335	0.8868	0.8446
55	18.07	9.033	0	4.516	3.613	3.011	2.581	2.258	2.007	1.807	1.642	1.505	1.39	1.29	1.204	1.129	1.063	1.004	0.9508	0.9033	0.8602
56	18.39	0	6.131	4.598	3.679	3.066	2.628	2.299	2.044	1.839	1.672	1.533	1.415	0	1.226	1.15	0	1.022	0.9681	0.9197	0.8759
57	18.72	9.361	0	4.681	3.744	3.12	2.675	2.34	2.08	1.872	1.702	1.56	1.44	1.337	1.248	1.17	1.101	1.04	0.9854	0.9361	0.8915
58	19.05	9.525	6.35	4.763	3.81	3.175	2.722	2.381	2.117	1.905	1.732	1.588	1.465	1.361	1.27	1.191	1.121	1.058	1.003	0.9525	0.9072
59	19.38	9.69	6.46	4.845	0	3.23	2.768	2.422	2.153	1.938	1.762	1.615	0	1.384	1.292	1.211	1.14	0	1.02	0.969	0.9228
60	19.71	9.854	6.569	4.927	3.942	3.285	2.815	2.463	2.19	1.971	1.792	1.642	1.516	1.408	1.314	0	1.159	1.095	1.037	0.9854	0.9385
61	20.04	10.02	6.679	5.009	4.007	0	2.862	0	0	2.004	1.821	1.67	1.541	1.431	0	1.252	1.179	1.113	1.055	1.002	0
62	20.36	10.18	6.788	5.091	4.073	3.394	2.909	2.546	0	2.036	1.851	1.697	1.566	1.455	1.358	0	1.198	1.131	1.072	1.018	0.9697
63	20.69	10.35	6.898	5.173	4.139	3.449	2.956	0	2.299	2.069	1.881	1.724	1.592	1.478	1.38	1.293	1.217	1.15	1.089	1.035	0.9854

Results (cont.)

For $l-1$ magic, error as K varies is relatively smooth (i.e., predicts) nearby error at different M. This does not hold for fmincon. *See Figure 7.*

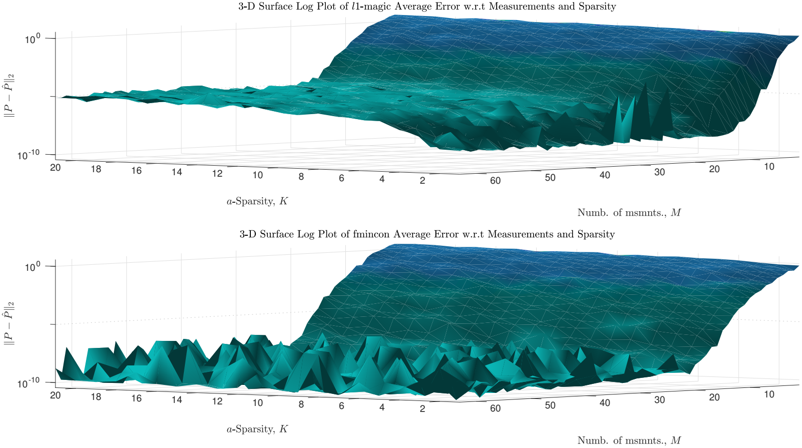
Fig. 7


3-D Log Plot of l1-magic Average Error w.r.t Sparsity
10^0
10^{-10}
$\|P - \hat{P}\|_2$ at every K
20
10
a-Sparsity, K
60
50
40
30
20
10
Numb. of msmnts., M
3-D Log Plot of fmincon Average Error w.r.t Sparsity
10^0
10^{-10}
$\|P - \hat{P}\|_2$ at every K
20
10
a-Sparsity, K
60
50
40
30
20
10
Numb. of msmnts., M

Results (cont.)

- On the whole, fmincon is superior to $l-1$ magic using default options for both. Notably, total computation time for **fmincon exceeded five hours, while $l-1$ magic required only about twenty-five minutes.** *See Figure 8.*
- Caveat: Don't be deceived by log plots! Error is indeed small for large M.

Fig. 8


3-D Surface Log Plot of l1-magic Average Error w.r.t Measurements and Sparsity
10^0
$\|P - \hat{P}\|_2$
10^{-10}
20 18 16 14 12 10 8 6 4 2
60 50 40 30 20 10
a-Sparsity, K
Numb. of msmnts., M
3-D Surface Log Plot of fmincon Average Error w.r.t Measurements and Sparsity
10^0
$\|P - \hat{P}\|_2$
10^{-10}
20 18 16 14 12 10 8 6 4 2
60 50 40 30 20 10
a-Sparsity, K
Numb. of msmnts., M

RESULTS (CONT.)

- In *Figure 9*, observe the very notable exceptions where the particularly expensive fmincon function loses to $l-1$ magic.

Fig. 9: (M, K) where fmincon loses (=1)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	0	0	0	1	0	1	0	1	0	0	0	0	0	0	1	0	0
6	1	1	1	1	1	0	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0
7	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	1	0	0	0
8	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
9	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0
11	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0
12	1	1	1	1	1	1	1	1	0	1	0	1	0	1	0	0	0	1	0	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0
14	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	0	0
15	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Summary of Findings

	K small	K large
M small	Quad. 1	Quad. 4
M large	Quad. 2	Quad. 3

- **Quad. 1:** Most surprising finding. Cheap conjugate gradients method ($l-1$ magic) is superior to very expensive fmincon function. Suggests problems in application require fine-tuned numerical schemes.

Summary of Findings

	K small	K large
M small	Quad. 1	Quad. 4
M large	Quad. 2	Quad. 3

- **Quad. 2:** Here, $l-1$ magic partly validates theory by relating sparsity of S to K (via coeff. ≈ 2.2) while fmincon cannot resolve even when given ample resources.

Summary of Findings

	K small	K large
M small	Quad. 1	Quad. 4
M large	Quad. 2	Quad. 3

- **Quad. 3:** "Inverse" of Quad 2. fmincon partly validates theory by picking up where $l-1$ magic fails. $l-1$ magic cannot resolve.

Summary of Findings

	K small	K large
M small	Quad. 1	Quad. 4
M large	Quad. 2	Quad. 3

- **Quad. 4:** Most expected finding. Both algorithms unstable (via std.dev. measure), problem essentially ill-posed.

Conclusion

Checklist for measuring:

- Measure randomly in as many dimensions as possible while avoiding noise
- Hope that your target data is sparse in these dimensions
- Custom-tailor your minimization algorithm to behave when given your measurements/sparsity
- Reconstruct and enjoy

Fig. 10: Primal Dual Method's Noise

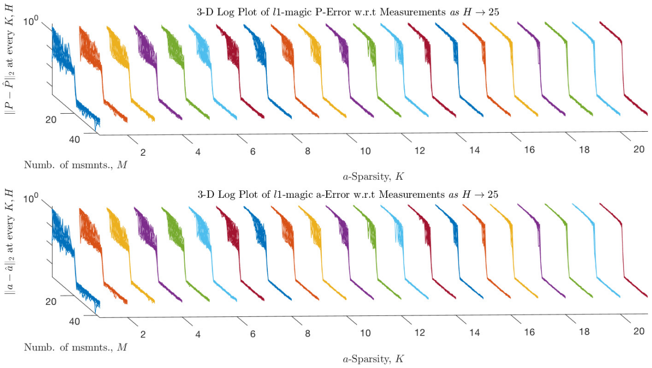


Fig. 11: Primal Dual Method's Avg. Success Indications

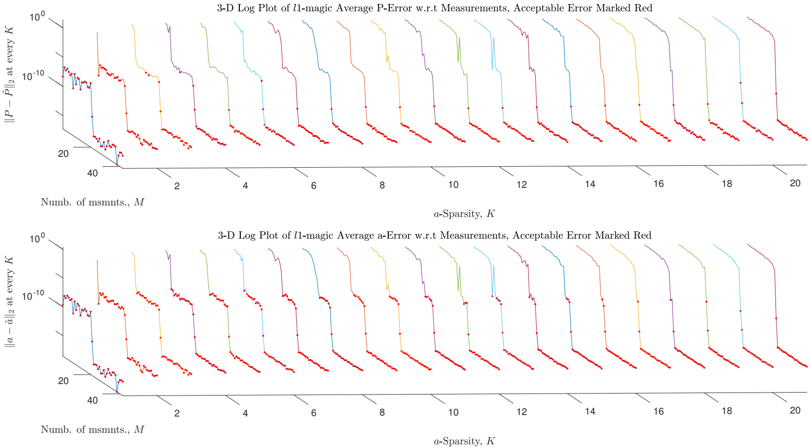


Fig. 12: Primal Dual Method's Smoothness w.r.t. Sparsity

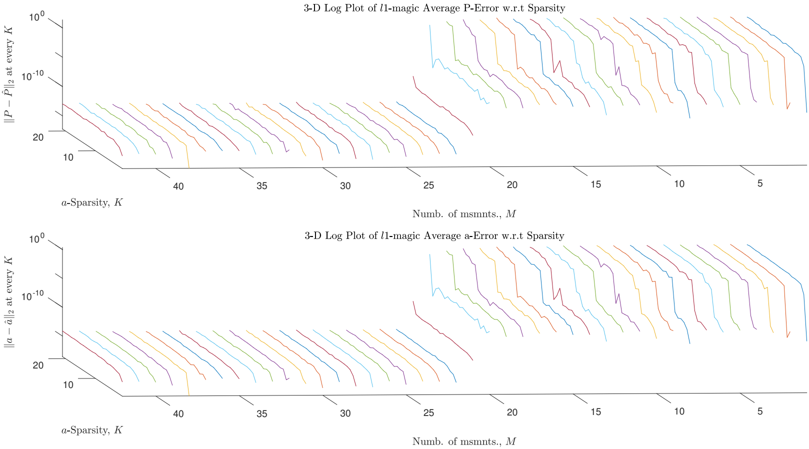
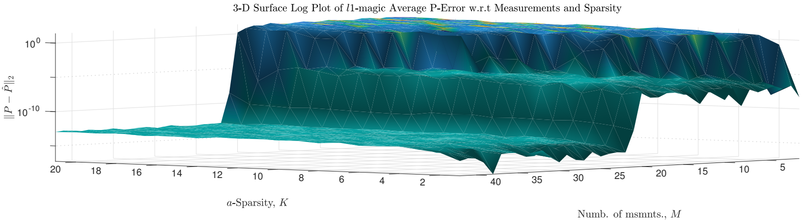
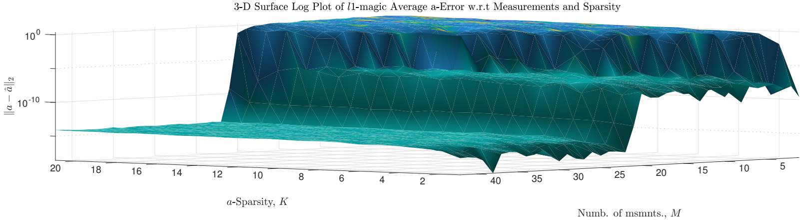


FIG. 13: PRIMAL DUAL METHOD'S ERROR COMPARISON





Conclusion

- Fixed sample space results indicate that, if possible, one should measure with respect to an optimal sample space.

END

Thank you for listening.